Stackelberg with multiple followers

An industry is composed of a dominant firm with costs $C(Q_d) = 32Q_d + Q_d^2$ and eight small competitive firms (identical) with costs $C(Q) = 70Q + 2Q^2$. The market demand is $Q = 100 - p$. Knowing that $Q = Q_f + Q_d$, determine the price and production equilibrium for each of the firms.

Solution

Cost structure of the monopolistic firm $C(Q_d) = 32Q_d + Q_d^2$
Cost structure of the competitive firms $C(Q) = 70Q + 2Q^2$

The competitive firms make decisions, taking into account the inverted market demand:

$$P = 100 - Q$$

Lay out the condition of perfect competition:

$$P = CMg$$

$$P = 70 + 4Q$$

Substitute prices in the market demand function:

$$100 - Q = 70 + 4Q_f$$

Substitute Q as:

$$100 - Q_d - 8Q_f = 70 + 4Q_f$$

Solve for the reaction function for the follower firms:

$$Q_f = \frac{30 - Q_d}{12}$$

Now I proceed to solve the maximization problem for the profit of the dominant firm:

$$\max_{Q_d} \Pi = P \cdot Q_d - C(Q_d)$$

Substitute:

$$\Pi_d = (100 - Q_d - 8Q_f)Q_d - 32Q_d - Q_d^2$$

Introduce the reaction function of the follower firms into this maximization, since the dominant firm chooses after the competitive firm:

$$\Pi_d = (100 - Q_d - \frac{30 - Q_d}{12})Q_d - 32Q_d - Q_d^2$$

$$\Pi_d = 48Q_d - \frac{1}{3}Q_d^2$$

Now I proceed to establish the first-order condition:

$$\frac{d\Pi_d}{dQ_d} = 0$$

Solving for Q_d

$$Q_d = 18$$

Now, I substitute the quantities that the dominant firm will produce into the reaction function of the competitive firm:

$$Q_f = 1$$

$$NQ_f = 8$$

Finally, calculate the market price:

$$P = 100 - Q$$

$$P = 74$$